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Paper

PAPER

MODELING NONLINEAR MAGNETOELASTIC BEHAVIOR OF THIN ELASTIC PLATES

Shohruh Safarov [1,*]

[1]Associate Professor of the Digital Technologies Alfraganus University Tashkent, Uzbekistan

* s.safarov@afu.uz

Abstract

This work investigates the mathematical framework for describing nonlinear magneto-electro-mechanical interactions in thin elastic plates of complex geometry. Special attention is given to the coupled behavior of electromagnetic fields and elastic responses, incorporating nonlinear constitutive relations and geometrically intricate boundaries. By applying Hamilton's variational principle, a unified system of nonlinear partial differential equations is derived, integrating mechanical and electromagnetic energy terms. Analytical methods combined with numerical simulations are employed to evaluate stress distributions, deformation modes, and multiphysical field coupling under varying boundary conditions and external excitations.

Key words: R-function, Hamilton-Ostragradskiy variation principle, Hooks law, Potential energy, Kinetic energy.

INTRODUCTION

Mathematical modeling of nonlinear processes in thin electromagnetic-elastic plates is essential for analyzing advanced engineering systems, including aerospace structures, piezoelectric sensors, and flexible electronics. The presence of complex geometries introduces challenges such as stress singularities, boundary effects, and nonlinear coupled interactions, necessitating modeling approaches that go beyond classical linear elasticity.

This study formulates and analyzes nonlinear coupled partial differential equations using variational principles, energy-based methods, and finite element approximations. The results provide a theoretical foundation for smart materials, adaptive structures, and multiphysics simulation tools. Key contributions in this field have been made by S.A. Ambartsumyan, G.E. Bagdasaryan, M.V. Belubekyan, V.L. Rvachev, L.V. Kurpa, I.T. Selezov, M.R. Korotkina, S.A. Nazarov, V.A. Kozlov, and Academicians V.Q. Qobulov, X.A. Rakhmatulin, and Professors Sh.A. Nazirov, T. Yuldashev, R. Indiaminov, and F.M. Nuraliev.

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DEVELOPMENT A MATHEMATICAL MODEL

A mathematical model describing the nonlinear deformation behavior of a magnetoelastic plate is developed based on Hamilton's variational principle. The formulation integrates the Kirchhoff–Love plate theory, Cauchy stress relations, and the Lorentz force interpreted within the framework of Hooke's law, while employing Maxwell's representation of the electromagnetic field tensor [1].

$$\int_t (\delta K - \delta\Pi + \delta A)\, dt = 0 \tag{1}$$

where: t is time, the product of variation; K is the kinetic energy; Π is the potential energy; A is the work of external volumetric and surface forces.

In accordance with the Kirchhoff–Love hypothesis, deformations along the thickness direction (Z-axis) of a thin plate are neglected, and the displacement components of the mid-surface are expressed as follows:

$$\begin{aligned} u_1 &= u(x,y,t) - z\frac{\partial w}{\partial x}, \\ u_2 &= v(x,y,t) - z\frac{\partial w}{\partial y}, \\ u_3 &= w(x,y,t) \end{aligned} \tag{2}$$

where: the displacement of the middle plane of the thin plate along the coordinate (x, y, z) axes.

At this stage, the variations of kinetic energy, potential energy, and external work are consolidated into the Hamilton–Ostrogradsky variational principle (1), which leads to a set of generalized equilibrium equations accompanied by the corresponding initial and boundary conditions.

$$\begin{cases} -\rho h\dfrac{\partial^2 u}{\partial t^2} + \dfrac{\partial N_{xx}}{\partial x} + \dfrac{\partial N_{xy}}{\partial y} + N_x + R_x + q_x + T_{zx} = 0, \\ -\rho h\dfrac{\partial^2 v}{\partial t^2} + \dfrac{\partial N_{xy}}{\partial x} + \dfrac{\partial N_{yy}}{\partial y} + N_y + R_y + q_y + T_{zy} = 0, \\ -\rho h\dfrac{\partial^2 w}{\partial t^2} + \dfrac{\partial^2 M_{xx}}{\partial x^2} + 2\dfrac{\partial^2 M_{xy}}{\partial x \partial y} + \dfrac{\partial^2 M_{yy}}{\partial y^2} \\ \quad + N_{xx}\dfrac{\partial w}{\partial x} + N_{yy}\dfrac{\partial w}{\partial y} + N_{xy}\dfrac{\partial^2 w}{\partial x \partial y} + \left(\dfrac{\partial N_{xx}}{\partial x} + \dfrac{\partial N_{xy}}{\partial y}\right)\dfrac{\partial w}{\partial x} \\ \quad + \left(\dfrac{\partial N_{yy}}{\partial y} + \dfrac{\partial N_{xy}}{\partial x}\right)\dfrac{\partial w}{\partial y} + N_z + R_z + q_z + T_{zz} = 0. \end{cases} \tag{3}$$

Natural limit conditions:

$$\begin{cases} (N_{xx} + N_{Px} + N_{Tx})\,\delta u\big|_x = 0, \qquad \left(N_{xy} + N_{Py} + N_{Txy}\right)\delta v\big|_x = 0, \\ M_{xx}\,\delta\dfrac{\partial w}{\partial x}\Big|_x = 0, \qquad M_{xy}\,\delta\dfrac{\partial w}{\partial y}\Big|_x = 0, \\ \left(N_{yy} + N_{Fy} + N_{Ty}\right)\delta v\big|_y = 0, \qquad \left(N_{xy} + N_{Fx} + N_{Tyx}\right)\delta u\big|_y = 0, \\ M_{yy}\,\delta\dfrac{\partial w}{\partial y}\Big|_y = 0, \qquad M_{xy}\,\delta\dfrac{\partial w}{\partial x}\Big|_y = 0, \\ \left[N_{xx}\dfrac{\partial w}{\partial x} + N_{xy}\dfrac{\partial w}{\partial y} - \dfrac{\partial M_{xx}}{\partial x} - \dfrac{\partial M_{xy}}{\partial y} + N_{Fz} + N_{Tzx}\right]\delta w\Big|_x = 0, \\ \left[N_{yy}\dfrac{\partial w}{\partial y} + N_{xy}\dfrac{\partial w}{\partial x} - \dfrac{\partial M_{yy}}{\partial y} - \dfrac{\partial M_{xy}}{\partial x} + N_{Fz} + N_{Tyz}\right]\delta w\Big|_y = 0. \end{cases}$$

COMPUTATIONAL ALGORITHM OF NUMERICAL SOLUTION OF THE PROBLEM

An algorithm for evaluating the geometrically nonlinear deformation behavior of electromagnetic thin plates [4] involves the following steps:

1. Formulation of solution functions that satisfy the prescribed boundary conditions.
2. Discretization of the governing equations with respect to spatial coordinates.
3. Solution of the resulting discrete system to determine the unknown variables within the solution framework.
4. Evaluation of the normal (transverse) displacements of the plate's mid-surface.

To compute the unknown variables in the motion equations according to the proposed algorithm, the displacement coefficients are determined using a combination of methods, including the variational Bubnov–Galerkin approach, Gaussian quadrature, the Newmark integration scheme, and iterative solution techniques.

The boundary (limit) equations for a magnetoelastic plate of complex geometry are constructed using the R-function method [3].

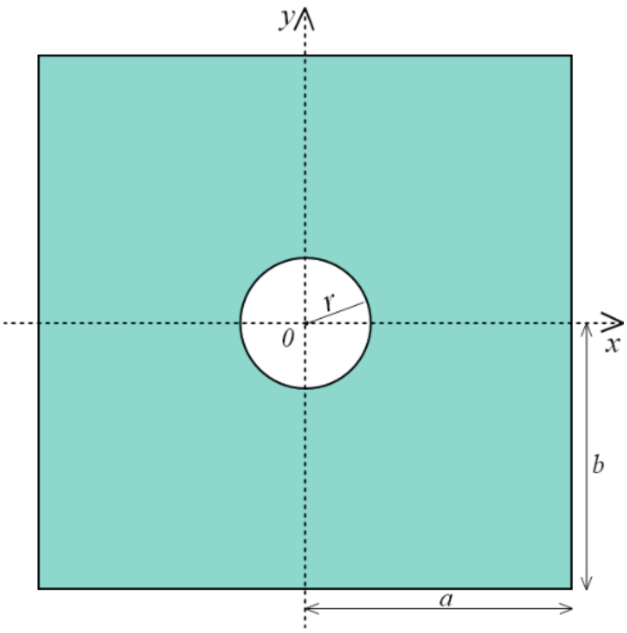

Figure 1. Magnetoelastic plate exhibiting a complex geometric configuration.

The iteration method is used to find the displacements $u_i(x,y,t), v_i(x,y,t), w_i(x,y,t)$ of the middle surface of the plate Y from the system of equations formed, and a numerical solution is obtained [6].

The effect of electromagnetic field forces on the process of geometric nonlinear deformation of a thin plate was analyzed by obtaining numerical values of $w_i(x,y,t)$ the displacement function when the thin plate is exposed to electromagnetic field forces and without taking into account the effect of field forces. At the same time, the difference between the maximum displacement points was 19This graph is shown through Figure 2 [7].

A numerical study was conducted to investigate the effect of thickness hhh on the deformation of a thin plate with complex geometry. Results (see Figure 3) show that decreasing hhh significantly increases bending deformations, reflecting greater flexibility and enhanced geometric nonlinearity in thinner plates.

The geometric and mechanical parameters, including plate dimensions, material properties,

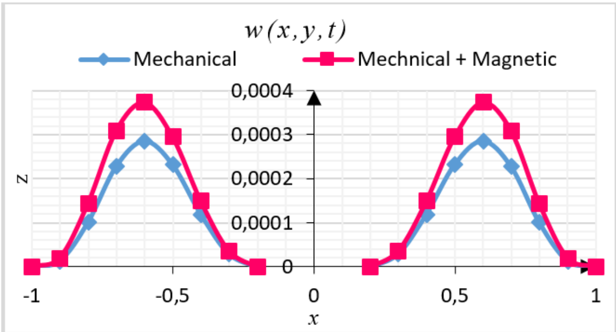

Figure 2. Effect of electromagnetic field forces on a thin plate with a complex geometry.

boundary conditions, and electromagnetic field characteristics, are presented below for reproducibility.

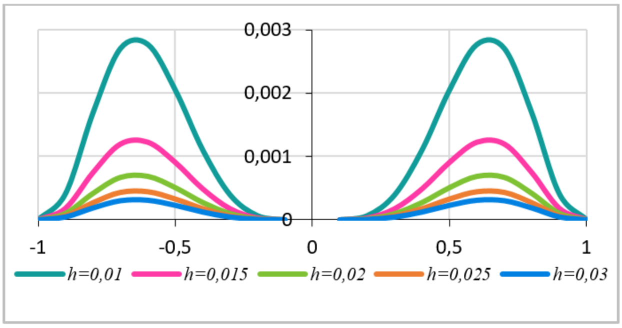

Figure 3. Study on the effects of plate thickness.

Numerical simulations and corresponding graphical illustrations (Figure 4) were carried out to investigate the effect of the inner radius r on the deformation behavior of a thin plate with a finely detailed complex geometry (see Figure 1). The computational experiments revealed that as the internal cut-out radius r decreases, the bending deformation of the plate under external loading conditions increases significantly [5].

The deformation behavior of a magnetoelastic thin plate with a complex geometry was analyzed under the influence of a second symmetric electromagnetic field configuration, as illustrated in Figure 4. Within this framework, comprehensive numerical simulations were carried out to evaluate the plate's nonlinear response to the applied field conditions.

The computational results, along with the corresponding deformation patterns and trends, are graphically illustrated in Figure 4. These findings provide valuable insights into the structural response of magnetoelastic plates under symmetric field distributions and serve as a basis for validating the effectiveness of the proposed modeling approach.

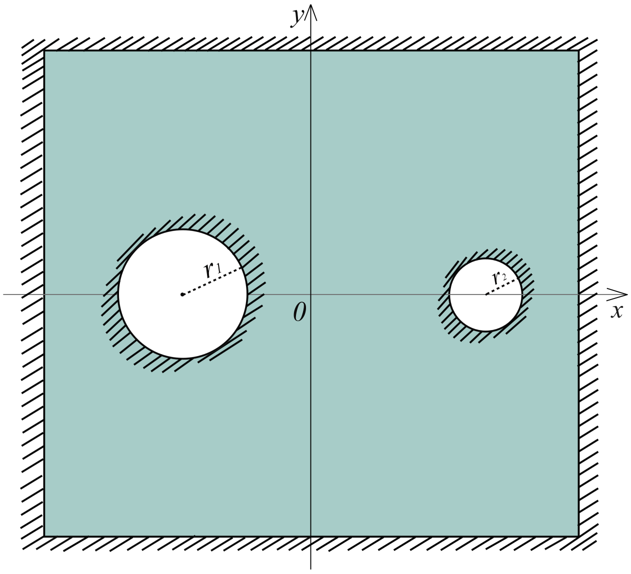

Figure 4. Magnetoelastic plate featuring asymmetrical and geometrically complex configurations.

An analytical expression for the applied asymmetric complex field, defined using the R-function method (see Figure 4), is presented in Equation (4). This formulation was employed to analyze the bending behavior of a magnetoelastic plate with an asymmetric, complex configuration. The resulting bending distribution along the coordinate axis is shown in Figure 5, clearly illustrating the deformation characteristics under the applied field [7].

$$\omega = (f_1 \wedge f_2) \wedge f_3 \wedge f_4 \qquad (4)$$

here

$$f_1 = \frac{a^2 - x^2}{2a} \geq 0, \qquad f_2 = \frac{b^2 - y^2}{2b} \geq 0,$$

$$f_3 = \frac{((x - a_1)^2 + y^2 - r^2)}{2r} \geq 0,$$

$$f_4 = \frac{((x + a_2)^2 + y^2 - r^2)}{2r} \geq 0.$$

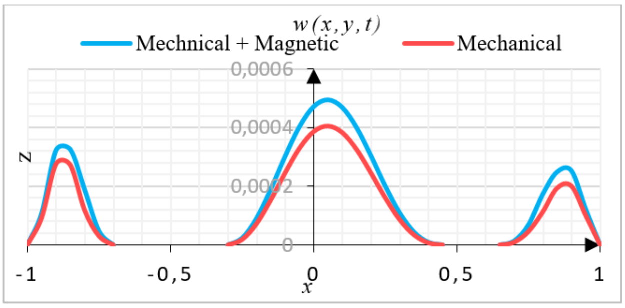

Figure 5. Bending behavior of magnetoelastic thin plates with asymmetrical complex geometries.

CONCLUSION

This research examines the geometrically nonlinear deformation behavior of thin plates with complex configurations subjected to electromagnetic field forces. Particular emphasis is placed on understanding how electromagnetic fields influence the nonlinear strain and stress states of magnetoelastic plates with nonstandard geometries.

A new mathematical framework has been established to describe the vibration characteristics of thin plates with intricate structural forms operating within an electromagnetic environment. In addition, efficient computational algorithms were developed to solve the governing equations, and a specialized software tool was implemented to perform numerical experiments.

For validation purposes, two types of complex-shaped plates—one symmetric and one asymmetric—were modeled and analyzed. The resulting numerical data were presented in tabular format and supplemented with graphical illustrations, enabling clearer interpretation of the deformation patterns and field interactions. The findings of this study form a solid foundation for addressing similar coupled nonlinear problems and offer practical value for engineering applications involving smart materials and multifunctional structural systems

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