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PAPER

BLOW-UP SOLUTIONS FOR A CLASS OF NONLINEAR PARABOLIC EQUATIONS NON-DIVERGENCE FORM

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Abstract

In this article, nonlinear equations of non-divergent parabolic type are considered. Solutions of nonlinear equations of non-divergent parabolic type with Blow-up properties under boundary conditions were also investigated. Hopf's maximum principle was used in the evaluation of the solutions. For nonlinear equations of non-divergent parabolic type, the conditions for the global existence of solutions over time and the existence of unbounded (blow-up) solutions are obtained. Top estimates of blow-up and global solutions of nonlinear equations of non-divergent parabolic type are shown. The properties of Blow-up solutions of a nonlinear parabolic equation of non-divergent form under boundary conditions were investigated, and estimates for explosion times were obtained in the problems of heat diffusion and combustion processes. In mathematical models describing the processes of heat diffusion and combustion in nonlinear media, the power of the source (absorption) is also common.

Key words: matematical model, asymptotic, combustion, nonlinear parabolic equation, self-similar solution, non-divergent form, Blow-up properties of solutions, global solutions.

INTRODUCTION

Today, research on equations describing the diffusion process is considered relevant and necessary worldwide and is widely applied in many fields of science and technology, in particular in applied mechanics, thermal physics, ecology, biophysics, biology, combustion processes, and

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other fields. Our scientists have shown that it is possible to construct mathematical models by expressing them using various complex mathematical equations. Most of the processes, such as salt-dust migration processes, heat transfer processes, filtration in loose soil, blood movement in small blood vessels, evaporation of waste, growth and migration of biological populations, are described by nonlinear partial differential equations of parabolic type. Therefore, the study of nonlinear mathematical models of non-divergent diffusion processes in nonlinear media, effective numerical solution schemes and algorithms.

ANALYSIS OF LITERATURE ON THE TOPIC

The first problems with the blow-up mode began in the mid-20th century with the burning theories of Zeldovich, Barenblatt, and Librovich. In the works of A.A.Samarskiy, A.S.Kalashnikov, V.A.Galaktionov, A.P.Mikhailov, B.I.Barenblatt, J.L.Lions, Daniela Giachetti, Pan Zheng, J. Vazgues, Ansgar Jüngel, L.Rossi, Juntang Ding and other scientists, the unboundedness of the solution in time (blow up), the phenomenon of finite heat propagation velocity and spatial localization of heat propagation, the properties of the existence of displacement in a finite time in nonlinear media under the influence of source and absorption were determined.

In our country, M.Aripov, Sh.Sadullaeva, A.Matyakubov, D.Mukhammadieva, Zh.Raimbekov, M.Khojimurodova were engaged in the study of new properties of mathematical models representing various processes and scientific works on non-divergent equations and systems of equations. Natural science based on self-similar analysis.

RESEARCH METHODOLOGY

In this article, we examine the blow-up characteristics of the following problem:

$$\frac{\partial u}{\partial t} = b(u)\left[\frac{\partial}{\partial x_1}\left(a(u)\frac{\partial u}{\partial x_1}\right) + \frac{\partial}{\partial x_2}\left(a(u)\frac{\partial u}{\partial x_2}\right)\right] + f(u),$$

(x,t)$\in D \times (0,T)$(1)

$$\frac{\partial u}{\partial n} + \sigma(x,t)u = 0, \qquad (x,t) \in \partial D \times (0,T)$$

$$u(x,0) = u_0(x) > 0, \qquad x \in \overline{D}$$

where D is a smooth bounded region R^2, $x = (x_1, x_2)$, $\overline{D}$ – closed region,

∇ – gradient sign, $\frac{\partial}{\partial n}$ – derivative with respect to the external normal.

In this work, we assume that the functions a, b, f are always positive on $C^2(R^+)$, σ function – non–negative, $C^1(\overline{Q_T})$, $(Q_T = D \times (0,T),\ R^+ = (0,+\infty))$, u_0 – positive function $C^3(\overline{D})$

on and

$$\frac{\partial u_0}{\partial n} + \sigma(x,t)u_0 = 0, \qquad (x,t) \in \partial D \times (0,T)$$

In the theory of nonlinear equations, the study of infinite solutions, in other words, blow-up solutions, is considered. Nonlinear problems with infinite solutions do not have a global solution over time: the solution grows infinitely over a certain time interval.

Let T be the maximum time of existence of the existing solution $u(x,t)$ (function). If $T < \infty$, then this solution will be infinite for a finite time, and a blow-up solution will occur. If $T = \infty$, we call the solution global.

ANALYSIS AND RESULTS

Let $u(x,t)$ $(u \in C^3(D \times (0,T)) \cap C^2(\overline{D} \times (0,T)))$ be the solution of problem (1) and $b(u) = u^{\frac{3}{2}}$, $a(u) = u^{\frac{1}{2}}$, $f(u) = u^2$. Then the following theorem is proven:

Theorem 1. Suppose the following conditions are met:

1. In $D \times (0,T)$ $\quad \sigma(x,t) \geq 0,\ \sigma_t(x,t) \leq 0$ (2)
2. $\beta = \min_{\overline{D}}\left\{\frac{\partial u_0}{\partial x_1}\left(u_0^{\frac{1}{2}}\frac{\partial u_0}{\partial x_1}\right) + \frac{\partial u_0}{\partial x_2}\left(u_0^{\frac{1}{2}}\frac{\partial u_0}{\partial x_2}\right) + u_0^{\frac{1}{2}}\right\} > 0$ (3)

Then this solution $u(x,t)$ (blow-up) exists for a finite time T and

$$u(x,t) \leq \frac{4}{\beta^2(T-t)^2}, \quad \text{where } T \leq \frac{2}{\beta}M_0^{\frac{1}{2}}, \quad M_0 = \max_{\overline{D}} u_0.$$

Proof. Consider the following function: $G = -u^{\frac{1}{2}}u_t + \beta u^2$.

From this

$$\max_{\overline{D}} G(x,0) = \max_{\overline{D}}\{-a(u_0)(b(u_0)\nabla(a(u_0)\nabla u_0) + f(u_0)) + \beta f(u_0)\} = 0 \quad (4)$$

is a bride.

In this domain $(x,t) \in \partial D \times (0,T)$

$$\frac{\partial G}{\partial n} = -\sigma\left(a'u + a\right)u_t + a\sigma u_t - \beta f'(u)\,\sigma u$$

will be.

From Hopp's maximum principle, the function G cannot have a maximum in the domain $\partial D \times (0,T)$, but its maximum in the domain $\overline{D} \times (0,T)$ is equal to 0.

Thus, we have $G \leq 0$ in domain $\overline{D} \times (0,T)$, and

$$u^{\frac{3}{2}} u_t \geq \beta \quad (5)$$

will be.

We integrate this equality (5)

$$\frac{1}{\beta}\int_{M_0}^{u(x_0,t)} s^{-\frac{3}{2}}\,ds \geq t$$

where at the point x_0 is $u_0(x_0) = M_0$.

Thus, the solution $u(x,t)$ exists for a finite time T, and the following equality holds:

$$T \leq \frac{1}{\beta}\int_{M_0}^{+\infty} s^{-\frac{3}{2}}\,ds$$

$$T \leq \frac{2}{\beta} M_0^{\frac{1}{2}},$$

We integrate the above inequality (5) with respect to $[t,s]$ $(0 < t < s < T)$, so that for each fixed x

$$H(u(x,t)) \geq H(u(x,t)) - H(u(x,s)) = \int_{u(x,t)}^{u(x,s)} s^{-\frac{3}{2}}\,ds = \int_t^s u^{-\frac{3}{2}} u_t\,dt \geq \beta(s-t)$$

we take the inequality.

So,

$$u(x,t) \leq H^{-1}(\beta(s-t))$$

where $H(z) = -\frac{2}{\sqrt{z}}$, $z > 0$ and H^{-1} are inverse H functions.

At the same time, we get $s \to T$:

$$u(x,t) \leq \frac{4}{\beta^2(T-t)^2}$$

And so, Theorem 1 is proven.

CONCLUSIONS

In this work, the properties of Blow-up solutions of a nonlinear parabolic equation of non-divergent form under boundary conditions were investigated, and estimates for explosion times were obtained in the problems of heat diffusion and combustion processes. The works of scholars dealing with such issues were analyzed [1-18]. Also, using Hopf's maximum principle, the conditions for the global existence of solutions over time and the existence of unbounded (blow-up) solutions for nonlinear equations of parabolic type were obtained, and top-level estimates of blow-up and global solutions were shown. In mathematical models describing the processes of heat diffusion and combustion in nonlinear media, the power of the source (absorption) was also studied in general terms depending on time, coordinates, and temperature.

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